2 Solving and rearranging

The equals sign is younger than the printing press. Robert Recorde introduced it in 1557, in an arithmetic book written in English, and explained his choice of two parallel lines of the same length: no two things could be more equal. Before that, writers spelled out “is equal to” every time. Recorde’s two lines carry one promise, and everything in this chapter keeps it: whatever you do to one side, you do to the other.

2.1 Undoing operations

To solve an equation, undo what was done to the unknown, in the reverse order, and do each step to both sides. Every operation has an **inverse operation** that undoes it: subtraction undoes addition, division undoes multiplication, a square root undoes a square.

Inverse operation
The operation that undoes another: subtraction undoes addition, division undoes multiplication.

Think of how the unknown was built. In $2.5x - 4 = 11$, someone took x, multiplied by 2.5, then subtracted 4. To get x back, reverse the steps: add 4 first, then divide by 2.5. Undoing in the wrong order, dividing by 2.5 first, is legal only if you divide every term, the 4 included, and that is where most errors happen.

Example 2.1

Solve $2.5x - 4 = 11$.

What is asked	The value of x that makes both sides equal.
Add	Add 4 to both sides: $2.5x = 15$.
Divide	Divide both sides by 2.5: $x = 6$.
Check	Put it back: $2.5 \times 6 - 4 = 15 - 4 = 11$.
Answer	$x = 6$

Undo the operations on the unknown in reverse order, and do every step to both whole sides.

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2.2 Rearranging a formula

Chemistry formulas are equations with letters in every place. **Rearranging** one means solving for one letter while the others stay letters. The moves are exactly the moves of the last section; the only difference is that you divide by V instead of by 2.5.

Rearranging
Solving a formula for one of its letters, leaving the others as letters.

Density is defined as $d = \frac{m}{V}$. To get the mass, undo the division by V: multiply both sides by V, and $m = dV$. To get the volume from there, divide both sides of $m = dV$ by d, and $V = \frac{m}{d}$. Three forms, one relationship.

Rearrange before you substitute numbers. A formula solved for the letter you want is easy to check by its units, it shows which way each quantity pushes the answer, and you can reuse it for every part of a problem. Plugging numbers in first buries the structure, and a slip in the arithmetic becomes impossible to find.

Isolate
Get the wanted letter alone on one side of the equation.

Example 2.2

The heat absorbed by a sample is $q = mc\Delta T$, where m is the mass, c the specific heat and ΔT the change in temperature. Solve for c, then find c when $q = 1{,}250\,\text{J}$, $m = 50.0\,\text{g}$ and $\Delta T = 12.5\,^\circ\text{C}$.

What is asked	c alone first, then its value with units.
Isolate	c is multiplied by m and by ΔT. Divide both sides by both: $c = \dfrac{q}{m\,\Delta T}$.
Substitute	$c = \dfrac{1250\,\text{J}}{(50.0\,\text{g})(12.5\,^\circ\text{C})}$
Calculate	$c = 2.00\,\text{J}/(\text{g}\,^\circ\text{C})$
Check	The units are joules per gram per degree, which is what a specific heat is. Put it back: $50.0 \times 2.00 \times 12.5 = 1250$.
Answer	$c = 2.00\,\text{J}/(\text{g}\,^\circ\text{C})$

Letters in the denominator

When the letter you want sits underneath a fraction bar, multiply both sides by it first to bring it up. From $d = \frac{m}{V}$: multiply by V to get $dV = m$, then divide by d. Trying to "flip" only part of an equation is the usual way the letter ends up in the wrong place.

When an equation is one fraction equal to another, $\frac{a}{b} = \frac{c}{d}$, you can **cross-multiply**: $ad = bc$. That is not a new rule; it is multiplying both sides by b and by d in one step. It works only when each side is a single fraction.

Cross-multiplying
For $\frac{a}{b} = \frac{c}{d}$, writing $ad = bc$. It is multiplying both sides by b and by d at once.

Isolate the letter you want before you put in numbers. Bring a letter out of a denominator by multiplying both sides by it.

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Practice set 2A 8 questions · 12 minutes · no calculator needed

1. Solve $4x + 7 = 31$.

(A) $x = 6$ **(B)** $x = 9.5$
(C) $x = 24$ **(D)** $x = 1.75$

2. Density is $d = \frac{m}{V}$. Which expression gives the volume?

(A) $V = dm$ **(B)** $V = \frac{d}{m}$
(C) $V = \frac{m}{d}$ **(D)** $V = m - d$

3. The relation $M_1V_1 = M_2V_2$ describes diluting a solution. Which expression gives V_2?

(A) $V_2 = \frac{M_1M_2}{V_1}$
(B) $V_2 = M_1V_1M_2$
(C) $V_2 = \frac{M_2}{M_1V_1}$
(D) $V_2 = \frac{M_1V_1}{M_2}$

4. Solve $\frac{12}{x} = \frac{3}{5}$.

(A) $x = 7.2$ **(B)** $x = 20$
(C) $x = 1.25$ **(D)** $x = 0.05$

5. Solve $3x + 8 = 5x - 2$.

(A) $x = 3$ **(B)** $x = 1.25$
(C) $x = -5$ **(D)** $x = 5$

6. A temperature is 59 °F. Using $T_F = 1.8\,T_C + 32$ and $T_K = T_C + 273$, what is it in kelvins?

(A) 288 K **(B)** 306 K
(C) 341 K **(D)** 258 K

7. For an ideal gas, $PV = nRT$. Solve for T, then find T when $P = 2$, $V = 6$, $n = 0.5$ and $R = 0.08$ (units omitted).

(A) $T = 0.0067$ **(B)** $T = 0.96$
(C) $T = 300$ **(D)** $T = 150$

8. A liquid mixture has an average density of 0.86 g/mL. It contains only liquid A (0.79 g/mL) and liquid B (0.99 g/mL). If x is the volume fraction of A, then $0.86 = 0.79x + 0.99(1 - x)$. What volume of A is in 500 mL of the mixture?

(A) 175 mL **(B)** 325 mL
(C) 250 mL **(D)** 65 mL

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3 Exponents and scientific notation

Around 230 BC Archimedes set out to count the grains of sand it would take to fill the universe. The Greek number system stopped at a myriad myriads, a hundred million, so before he could answer he had to invent a way of writing numbers by orders of magnitude. His answer was about 10^{63} grains. Chemistry has the same problem in both directions: a gram of water holds about 3×10^{22} molecules, and each one has a mass near 3×10^{-23} g. Scientific notation is how those numbers stay readable.

3.1 The exponent rules

An **exponent** counts factors: $10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$. Every exponent rule follows from that count, so if you forget one, write out a small case and count.

Exponent
The small raised number in a^n: how many times the base a is used as a factor.

Rule	In symbols	Example
Multiply: add	$a^m \cdot a^n = a^{m+n}$	$10^3 \cdot 10^4 = 10^7$
Divide: subtract	$a^m / a^n = a^{m-n}$	$10^2 / 10^5 = 10^{-3}$
Power of a power	$(a^m)^n = a^{mn}$	$(10^{-2})^3 = 10^{-6}$
Negative exponent	$a^{-n} = 1/a^n$	$10^{-3} = \frac{1}{1000} = 0.001$
Zero exponent	$a^0 = 1$	$10^0 = 1$

A **negative exponent** does not make a number negative. It puts the factors underneath: 10^{-3} is one thousandth, a small positive number. In chemistry, negative exponents describe the very small, such as the mass of an atom in grams or the concentration of an ion in an acid.

Negative exponent
A reciprocal: $a^{-n} = \frac{1}{a^n}$. It makes a number small, not negative.

Multiplying powers of ten adds exponents, dividing subtracts them, and a negative exponent means a reciprocal.

3.2 Scientific notation

A number in **scientific notation** is a coefficient between 1 and 10 times a power of ten:

$$a \times 10^n, \qquad 1 \le a < 10$$

Scientific notation
A number written as $a \times 10^n$ with $1 \le a < 10$.

The exponent says how many places the decimal point moved. Moving it left to make a big number small gives a positive exponent; moving it right to make a small number bigger gives a negative one. So $45{,}000 = 4.5 \times 10^4$ and $0.00072 = 7.2 \times 10^{-4}$.

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A quick check settles the sign every time. A number larger than 10 has a positive exponent, and a number smaller than 1 has a negative exponent. A number between 1 and 10 has exponent 0.

Comparing numbers in scientific notation, look at the exponent first. 9.9×10^{-5} is smaller than 1.1×10^{-4} even though 9.9 is bigger than 1.1, because 10^{-5} is a tenth of 10^{-4}. Only when the exponents match do the coefficients decide.

Order of magnitude
The power of ten nearest a number. 3.2×10^4 and 8.9×10^4 share an exponent but differ by almost a factor of 3.

Making a coefficient legal

After a calculation the coefficient is often out of range: 34.5×10^6, or 0.27×10^{-3}. Fix it by moving the decimal in the coefficient and adjusting the exponent the opposite way, so the value does not change. Making the coefficient smaller by a factor of 10 makes the power larger by one: $34.5 \times 10^6 = 3.45 \times 10^7$. Making it larger lowers the power: $0.27 \times 10^{-3} = 2.7 \times 10^{-4}$.

Example 3.1

Write $0.000\,060\,2$ and $602{,}000{,}000$ in scientific notation, and say which is larger by how many powers of ten.

What is asked	Two numbers in $a \times 10^n$ form, then the difference in their exponents.
Small one	Move the decimal 5 places right to get 6.02: 6.02×10^{-5}.
Large one	Move the decimal 8 places left: 6.02×10^8.
Compare	Same coefficient, so the ratio is $10^8 / 10^{-5} = 10^{13}$.
Check	The first is under 1, so its exponent must be negative; the second is huge, so positive.
Answer	6.02×10^{-5} **and** 6.02×10^8**; the second is** 10^{13} **times larger**

The exponent counts the places the decimal moved. Bigger than 10, positive; smaller than 1, negative.

3.3 Multiplying and dividing

To multiply, multiply the coefficients and add the exponents. To divide, divide the coefficients and subtract the exponents. Then make the coefficient legal.

$$(3.0 \times 10^4)(5.0 \times 10^{-7}) = 15 \times 10^{-3} = 1.5 \times 10^{-2}$$

$$\frac{6.0 \times 10^{23}}{1.5 \times 10^{-2}} = 4.0 \times 10^{25}$$

Watch the subtraction of a negative exponent: $23 - (-2) = 25$. Losing that sign is the most common error in this section, and it changes the answer by a factor of 10^4.

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Practice set 3A 8 questions · 12 minutes · no calculator needed

1. What is $0.000\,48$ in scientific notation?

(A) 4.8×10^{4} **(B)** 4.8×10^{-4}
(C) 48×10^{-5} **(D)** 4.8×10^{-3}

2. What is $10^{3} \times 10^{-5}$?

(A) 10^{-15} **(B)** 10^{8}
(C) 10^{-2} **(D)** 10^{2}

3. Which number is the smallest?

(A) 9.9×10^{-5} **(B)** 1.1×10^{-4}
(C) 5.0×10^{-4} **(D)** 2.0×10^{-3}

4. What is $(3.0 \times 10^{4})(5.0 \times 10^{-7})$?

(A) 15×10^{-3} **(B)** 1.5×10^{-3}
(C) 1.5×10^{-2} **(D)** 1.5×10^{-28}

5. What is $\dfrac{8.0 \times 10^{6}}{2.0 \times 10^{-3}}$?

(A) 4.0×10^{3} **(B)** 4.0×10^{9}
(C) 4.0×10^{-9} **(D)** 1.6×10^{4}

6. What is $4.2 \times 10^{3} + 6.0 \times 10^{2}$?

(A) 1.02×10^{6} **(B)** 10.2×10^{5}
(C) 4.8×10^{3} **(D)** 1.02×10^{4}

7. A sample contains 1.2×10^{22} atoms. One mole is 6.0×10^{23} atoms. How many moles is the sample, and how many atoms would 10.0 such samples contain?

(A) 50 mol; 1.2×10^{23} atoms
(B) 0.020 mol; 1.2×10^{32} atoms
(C) 0.20 mol; 1.2×10^{23} atoms
(D) 0.020 mol; 1.2×10^{23} atoms

8. What is $\sqrt{(2.0 \times 10^{-3})(8.0 \times 10^{-4})}$?

(A) 4.0×10^{-4} **(B)** 1.3×10^{-3}
(C) 4.0×10^{-3} **(D)** 1.6×10^{-6}

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4 Significant figures and rounding

When the Great Trigonometrical Survey of India finished its calculations for the highest mountain it had measured, the average came out at exactly 29,000 feet. It was published as 29,002, the story goes, because a round number would have looked like a guess. Whether or not the story is true, the worry behind it is real: the digits you write make a claim about how well you measured. Significant figures are the rules for making that claim honestly.

4.1 Measured and exact numbers

Every **measured number** carries some uncertainty, and the way you write it shows how much. On a ruler marked in millimeters you can read a length as 23.4 mm: the 23 is certain and the 4 is your estimate between two marks. Writing 23.43 mm would claim a precision the ruler cannot give, and writing 23 mm would throw away a digit you did measure.

The digits you write, the certain ones and the one estimated digit, are the **significant figures**. Record every measurement with exactly one estimated digit. On a graduated cylinder marked every 1 mL, a reading is written to the tenth: 36.5 mL.

An **exact number** has no uncertainty. Counted things are exact: 12 eggs, 3 trials, 2 hydrogen atoms in a water molecule. So are definitions: $1\ \text{m} = 100\ \text{cm}$ and $1\ \text{in} = 2.54\ \text{cm}$ exactly. Exact numbers never limit the significant figures of an answer.

Record a measurement with every certain digit plus one estimated digit. Counted and defined numbers are exact.

Measured number
A value read from an instrument. Its last digit is an estimate.

Significant figures
The certain digits of a measurement plus the one estimated digit.

Exact number
A counted value or a defined relationship. It has unlimited significant figures.

4.2 Counting significant figures

Nonzero digits always count. Zeros are the only digits that need rules, and each rule asks whether the zero was measured or is only holding a place for the decimal point.

Zeros	Count?	Example
Between nonzero digits	Yes	5008 has 4; 3.07 has 3
Leading, before the first nonzero	No	0.0072 has 2
Trailing, with a decimal point	Yes	2.500 has 4; 120. has 3
Trailing, no decimal point	Unclear	1200 has 2, 3 or 4

A **leading zero** only locates the decimal point. 0.0072 g is 7.2 mg; change the unit and the zeros disappear, which shows they were never measured. A trailing zero after a decimal

Leading zero
A zero before the first nonzero digit. It only places the decimal point and never counts.

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point is the opposite: nobody writes 2.500 unless the instrument read that last zero, so it counts.

A number such as 1200 is ambiguous. Scientific notation removes the doubt, because every digit in the coefficient counts: 1.2×10^{3} has two, 1.200×10^{3} has four. When you need to show trailing zeros as measured, write the number in scientific notation.

Example 4.1

How many significant figures are in each: 0.04050 g, 3.0060 L and 6.020×10^{-4} m?

What is asked A count for each of three numbers.

First In 0.04050 the leading 0.0 do not count. 4, the zero between 4 and 5, 5, and the trailing zero after the decimal do: 4.

Second In 3.0060 every digit counts: the zeros are between nonzero digits or trailing after a decimal point: 5.

Third In 6.020×10^{-4} count the coefficient only: 4.

Answer **4, 5 and 4**

Nonzero digits count. Zeros count when they sit between digits or trail after a decimal point. Leading zeros never count.

4.3 Rounding

To round, look only at the first digit you drop. If it is 5 or more, raise the last kept digit by one; if it is less than 5, leave it. 2.3449 rounded to three figures is 2.34, because the first dropped digit is 4. Rounding in two stages, first to 2.345 and then to 2.35, is a mistake: round once, from the full number.

Rounding a whole number to fewer figures replaces dropped digits with zeros, and those zeros are placeholders: 48,720 to two figures is 49,000, best written 4.9×10^{4}.

Rounding
Dropping digits from the right. If the first dropped digit is 5 or more, the last kept digit goes up by one.

4.4 Calculating with measurements

An answer cannot be more precise than the measurements it came from. Two rules decide how far to round, and they depend on the operation.

The **multiplication rule**: for multiplication and division, the answer keeps as many significant figures as the measurement with the fewest. A density from 25.36 g and 12.1 mL has three: $\frac{25.36}{12.1}=2.0959\ldots$, which rounds to 2.10 g/mL. The trailing zero is required; 2.1 would claim only two figures.

Multiplication rule
In a product or quotient, keep as many significant figures as the measurement with the fewest.

The **addition rule**: for addition and subtraction, count decimal places, not significant figures. The answer stops at the last decimal place that every number has. $15.2+0.355+8.04=23.595$, and 15.2 is known only to the tenths, so the sum is 23.6.

Addition rule
In a sum or difference, keep as many decimal places as the measurement with the fewest.

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Practice set 4A 8 questions · 12 minutes · calculator allowed

1. How many significant figures are in 0.003 040 g?

(A) 3 **(B)** 4
(C) 6 **(D)** 7

2. Which measurement has exactly three significant figures?

(A) 0.0250 L **(B)** 2.500 L
(C) 250.0 L **(D)** 0.025 L

3. What is 0.006 748 5 rounded to three significant figures?

(A) 0.006749 **(B)** 0.00674
(C) 0.00675 **(D)** 0.006

4. A student measures 12.0 mL, 3.45 mL and 0.218 mL of three liquids and pours them together. How should she report the total volume?

(A) 16 mL **(B)** 15.7 mL
(C) 15.67 mL **(D)** 15.668 mL

5. What is $\dfrac{4.0 \times 3.256}{1.10}$ to the correct number of significant figures?

(A) 11.84 **(B)** 11.8
(C) 11 **(D)** 12

6. A block measures 2.15 cm by 4.0 cm by 1.025 cm and has a mass of 24.71 g. What is its density, to the correct number of significant figures?

(A) 2.8 g/cm^3 **(B)** 2.80 g/cm^3
(C) 2.803 g/cm^3 **(D)** 3 g/cm^3

7. A flask weighs 48.27 g empty and 73.4 g with a liquid in it. The liquid's volume is 24.6 mL. What is the density of the liquid?

(A) 2.98 g/mL **(B)** 1.0 g/mL
(C) 1.021 g/mL **(D)** 1.02 g/mL

8. Three trials give masses of 2.315 g, 2.31 g and 2.3226 g. The student averages them and divides the average by the exact number 2. How should the result be reported?

(A) 1.1579 g **(B)** 1.2 g
(C) 1.16 g **(D)** 1.158 g

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5 Units and dimensional analysis

In July 1983 an Air Canada Boeing 767 ran out of fuel at 41,000 feet over northwestern Ontario. The fuel had been checked; the arithmetic had been done. It had been done with 1.77, the number of pounds in a liter of fuel, where the new metric airplane needed 0.80, the number of kilograms. The crew loaded less than half the fuel they needed and glided the airplane to a landing on a disused runway at Gimli. Nobody was badly hurt. Every number in that calculation was right except its unit.

5.1 Units and metric prefixes

A number in chemistry is never just a number. 25 means nothing; 25 g, 25 mL and 25 °C are three different facts. Carry the **unit** with every number through every step, and write it on every line. The units will tell you whether a setup is right before you touch a calculator.

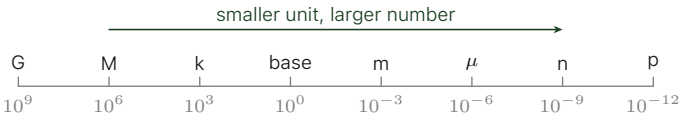
Unit
The label that says what a number counts: grams, liters, seconds.

The metric system builds every size of unit from one base unit and a **metric prefix**. A prefix is a power of ten, and that is all it is: a milligram is 10^{-3} g and a kilometer is 10^{3} m. The prefixes you will use every week are these.

Metric prefix
A word in front of a unit that multiplies it by a power of ten: kilo- is 10^{3}, milli- is 10^{-3}.

Prefix	Symbol	Factor	One of it is
giga-	G	10^{9}	$1\,\text{Gm} = 10^{9}\,\text{m}$
mega-	M	10^{6}	$1\,\text{Mg} = 10^{6}\,\text{g}$
kilo-	k	10^{3}	$1\,\text{kg} = 10^{3}\,\text{g}$
centi-	c	10^{-2}	$1\,\text{cm} = 10^{-2}\,\text{m}$
milli-	m	10^{-3}	$1\,\text{mL} = 10^{-3}\,\text{L}$
micro-	μ	10^{-6}	$1\,\mu\text{g} = 10^{-6}\,\text{g}$
nano-	n	10^{-9}	$1\,\text{nm} = 10^{-9}\,\text{m}$
pico-	p	10^{-12}	$1\,\text{pm} = 10^{-12}\,\text{m}$

The safest way to write a prefix as a conversion factor is with the prefixed unit as 1, as in the last column. $1\,\text{nm} = 10^{-9}\,\text{m}$ is hard to get backward. Writing it the other way, $1\,\text{m} = 10^{9}\,\text{nm}$, is also true, but the exponent's sign flips, and that flip is where most prefix errors come from.



The prefixes in steps of a thousand. Moving to a smaller unit makes the number larger: 1 g is 1000 mg.

A prefix is a power of ten. Write each one with the prefixed unit as 1: $1\,\text{mg} = 10^{-3}\,\text{g}$.

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5.2 Conversion factors

A **conversion factor** is a fraction whose top and bottom are the same quantity in different units. Since $1\,\text{L} = 1000\,\text{mL}$, both $\frac{1000\,\text{mL}}{1\,\text{L}}$ and $\frac{1\,\text{L}}{1000\,\text{mL}}$ equal 1. Multiplying by 1 does not change a quantity, only the unit it is written in. This is the same fact from Chapter 1 that lets you simplify a fraction.

Conversion factor
A ratio of two equal quantities in different units, such as $\frac{1000\,\text{mL}}{1\,\text{L}}$. Its value is 1.

Dimensional analysis is choosing which way up to write each factor. Units cancel like numbers: a unit on top cancels the same unit underneath. So write the factor with the unit you want to get rid of on the bottom.

$$250\,\text{mL} \times \frac{1\,\text{L}}{1000\,\text{mL}} = 0.250\,\text{L}$$

Dimensional analysis
Converting by multiplying by conversion factors arranged so the units you do not want cancel.

If the factor were upside down you would get $250{,}000\,\text{mL}^2/\text{L}$, and the unit itself would tell you the setup is wrong. That is the whole value of the method: a wrong setup produces a wrong unit, which you can see.

Example 5.1

A vitamin tablet contains $450\,\mu\text{g}$ of folic acid. How many milligrams is that?

What is asked	Milligrams. There is no single factor from micrograms to milligrams in the table, so go through grams.
Set up	$450\,\mu\text{g} \times \frac{10^{-6}\,\text{g}}{1\,\mu\text{g}} \times \frac{1\,\text{mg}}{10^{-3}\,\text{g}}$
Cancel	μg cancels in the first factor, g in the second; mg is left.
Calculate	$450 \times 10^{-6} \div 10^{-3} = 450 \times 10^{-3} = 0.450\,\text{mg}$
Check	A milligram is larger than a microgram, so the number must get smaller. It did, by a factor of 1000.
Answer	**0.450 mg**

Write each factor so the unit you want to cancel is on the bottom. If the units do not cancel to the one you want, the setup is wrong.

5.3 Chains of factors

Most conversions take more than one factor. Line them up in one row, each factor cancelling the unit left by the one before, and do the arithmetic once at the end. A speed in kilometers per hour converted to meters per second needs two factors, one for each unit:

$$90\,\frac{\text{km}}{\text{h}} \times \frac{1000\,\text{m}}{1\,\text{km}} \times \frac{1\,\text{h}}{3600\,\text{s}} = 25\,\frac{\text{m}}{\text{s}}$$

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Practice set 5A 8 questions · 14 minutes · calculator allowed

1. How many milliliters are in 0.0250 L?

(A) 0.0000250 mL **(B)** 2.50 mL
(C) 25.0 mL **(D)** 250 mL

2. Which conversion factor is written correctly?

(A) 1 cm $= 10^{2}$ m **(B)** 1 kg $= 10^{3}$ g
(C) 1 mg $= 10^{3}$ g **(D)** 1 nm $= 10^{9}$ m

3. A sample has a mass of 3.2 mg. What is its mass in micrograms?

(A) 3.2 × 10^{6} µg **(B)** 320 µg
(C) 3200 µg **(D)** 0.0032 µg

4. A volume is 4.5 cm^{3}. How many cubic meters is that?

(A) 4.5×10^{6} m^{3}
(B) 4.5×10^{-6} m^{3}
(C) 4.5×10^{-4} m^{3}
(D) 4.5×10^{-2} m^{3}

5. Mercury has a density of 13.6 g/mL. What is the mass of 2.50 mL of mercury?

(A) 0.184 g **(B)** 5.44 g
(C) 34.0 g **(D)** 16.1 g

6. A car travels at 65 miles per hour. What is its speed in meters per second? (1 mi $= 1609$ m)

(A) 29 m/s **(B)** 1.7 × 10^{3} m/s
(C) 1.0 × 10^{5} m/s **(D)** 0.040 m/s

7. A swimming pool holds 450 m^{3} of water. The density of water is 1.00 g/mL. What is the mass of the water in kilograms?

(A) 4.5×10^{2} kg **(B)** 4.5×10^{5} kg
(C) 4.5×10^{8} kg **(D)** 4.5×10^{3} kg

8. A solution contains 0.250 mol of solute per liter. A student needs 3.75×10^{-3} mol of solute. Each drop from her pipet is 0.050 mL. How many drops does she need?

(A) 0.75 drops **(B)** 3000 drops
(C) 300 drops **(D)** 15 drops

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6 Proportional reasoning

In 1638 Galileo explained why nature cannot build giants by simply enlarging ordinary animals. Make an animal twice as tall in every direction and its weight grows eight times, because weight follows volume, but the strength of its bones grows only four times, because strength follows the area of a bone's cross-section. Scale it up far enough and it breaks under its own weight. Galileo needed no measurements for this, only the shape of each relationship: how one quantity responds when another changes.

6.1 Direct proportion

Two quantities are **directly proportional** when their ratio never changes:

$$y = kx, \qquad \frac{y}{x} = k$$

The number k is the **constant of proportionality**. Double x and y doubles; cut x to a third and y is cut to a third. The mass of a sample of one substance is directly proportional to its volume, with density as the constant, and the mass of a pure substance is proportional to the number of moles, with the molar mass as the constant.

A direct proportion can be solved two ways. Find the constant first, $k = \frac{y}{x}$, and multiply; or set up equal ratios, $\frac{y_1}{x_1} = \frac{y_2}{x_2}$, and solve. The second way skips the constant and is faster when you only need one new value.

Directly proportional
Two quantities whose ratio is constant: $y = kx$. Double one and the other doubles.

Constant of proportionality
The fixed number k in $y = kx$ or $y = \frac{k}{x}$.

Example 6.1

2.00 g of a salt dissolves to make 50.0 mL of a solution. At the same concentration, how much salt is needed for 175 mL?

What is asked	A mass of salt, for a new volume at the same ratio.
Equal ratios	$\dfrac{2.00\,\text{g}}{50.0\,\text{mL}} = \dfrac{x}{175\,\text{mL}}$
Solve	$x = 2.00\,\text{g} \times \dfrac{175}{50.0} = 7.00\,\text{g}$
Check	175 mL is 3.5 times 50.0 mL, so the salt must be 3.5 times 2.00 g.
Answer	**7.00 g**

Directly proportional means a constant ratio. Scale both quantities by the same factor.

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6.2 Inverse proportion

Two quantities are **inversely proportional** when their product never changes:

$$y = \frac{k}{x}, \qquad xy = k$$

Inversely proportional
Two quantities whose product is constant: $y = \frac{k}{x}$. Double one and the other halves.

Double x and y halves; triple x and y falls to a third. The time a trip takes is inversely proportional to the speed. In the course you will see the pressure and volume of a gas at constant temperature behave this way, written $P_1V_1 = P_2V_2$.

For an inverse proportion the useful equation is equal products, $x_1y_1 = x_2y_2$. Students often set up equal ratios out of habit, which gives an answer that moves the wrong way. Before calculating, decide in words whether the new value should be larger or smaller.

Example 6.2

A gas sample has volume 6.0 L at pressure 1.5 atm. Pressure and volume are inversely proportional here. What is the volume at 4.5 atm?

What is asked	The new volume.
Direction	Pressure triples, from 1.5 to 4.5, so volume must fall to a third.
Equal products	$1.5 \times 6.0 = 4.5 \times V_2$, so $V_2 = \dfrac{9.0}{4.5} = 2.0\,\text{L}$
Check	$6.0 \div 3 = 2.0$. Equal ratios would have given 18 L, which moves the wrong way.
Answer	**2.0 L**

Inversely proportional means a constant product. Say in words which way the answer must move before you compute it.

6.3 Proportional to a power

Many relationships use a power: an area grows with the square of a length, a volume with the cube. Here the **scale factor** is raised to the same power. If every length of a cube doubles, its surface area grows by $2^2 = 4$ and its volume by $2^3 = 8$.

Scale factor
The number by which a quantity is multiplied, such as 2 for a doubling.

Relationship	If x doubles	If x is multiplied by 3
$y = kx$	y doubles	y triples
$y = kx^2$	y multiplies by 4	y multiplies by 9
$y = kx^3$	y multiplies by 8	y multiplies by 27
$y = \dfrac{k}{x}$	y halves	y falls to $\frac{1}{3}$
$y = \dfrac{k}{x^2}$	y falls to $\frac{1}{4}$	y falls to $\frac{1}{9}$

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Practice set 6A 8 questions · 12 minutes · calculator allowed

1. A recipe uses 3.0 g of baking soda for every 250 mL of water. How much baking soda goes with 600 mL of water?

(A) 1.25 g **(B)** 7.2 g
(C) 50 g **(D)** 6.0 g

2. At constant temperature, a gas's pressure and volume are inversely proportional. A sample at 2.0 atm occupies 9.0 L. What is its volume at 3.0 atm?

(A) 6.0 L **(B)** 13.5 L
(C) 4.5 L **(D)** 0.67 L

3. If y is proportional to x^2, what happens to y when x is multiplied by 3?

(A) It is multiplied by 3
(B) It is multiplied by 6
(C) It is multiplied by 9
(D) It is multiplied by 27

4. If $y = \dfrac{k}{x^2}$ and x doubles, what happens to y?

(A) It is multiplied by 4 **(B)** It falls to one fourth
(C) It halves **(D)** It doubles

5. A 12.0 g sample of a pure substance contains 0.200 mol. How many moles are in 45.0 g of it?

(A) 0.0533 mol **(B)** 0.200 mol
(C) 0.750 mol **(D)** 9.00 mol

6. In $y = \dfrac{k\,a\,b^2}{c}$, a is halved, b doubles and c doubles. By what factor does y change?

(A) 4 **(B)** $\frac{1}{2}$
(C) 2 **(D)** 1

7. For a gas, $P = \dfrac{nRT}{V}$. The temperature rises from 300 K to 450 K, and the volume is reduced to one third. A sample started at 1.2 atm. What is the new pressure?

(A) 0.60 atm **(B)** 1.8 atm
(C) 5.4 atm **(D)** 2.4 atm

8. A cube of metal has a mass of 8.0 g. A second cube of the same metal has edges 2.5 times as long. A third cube's volume is half the second's. What is the mass of the third cube?

(A) 125 g **(B)** 63 g
(C) 25 g **(D)** 10 g

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7 Logarithms

John Napier spent about twenty years computing a table of numbers by hand and published it in 1614. With it, a multiplication became an addition: look up the logarithms of two numbers, add them, and look up the number whose logarithm is the sum. Astronomers who had lost months to long multiplications adopted the tables almost at once. You will not use a table, but you will use the idea every time you meet pH: a logarithm turns a number with an awkward power of ten into an ordinary one.

7.1 What a logarithm is

A **logarithm** answers one question: what power of ten is this number? Since $10^3 = 1000$, the logarithm of 1000 is 3. Written as a pair of statements,

$$\log x = y \quad \text{means} \quad 10^y = x$$

So $\log 100 = 2$, $\log 10 = 1$, $\log 1 = 0$, $\log 0.1 = -1$ and $\log 0.001 = -3$. A logarithm is an exponent, and every fact about logarithms is a fact about exponents in different clothing.

Two consequences are worth stating plainly. The logarithm of a number between 0 and 1 is negative, and the logarithm of zero or of a negative number does not exist, because no power of ten is zero or negative. A negative logarithm is normal; a negative number inside a logarithm means something has gone wrong.

Logarithm
$\log x$ is the power to which 10 must be raised to give x.

Example 7.1

Without a calculator, find $\log 10^{-5}$, $\log 0.01$ and $\log 1{,}000{,}000$.

What is asked	Each number's power of ten.
10^{-5}	It is already a power of ten: -5.
0.01	$0.01 = 10^{-2}$, so -2.
1,000,000	10^6, so 6.
Answer	**-5, -2 and 6**

A logarithm is an exponent: $\log x$ is the power of ten that makes x.

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7.2 Numbers between the powers

Most numbers are not whole powers of ten. The logarithm of 3000 lies between 3 and 4, because 3000 lies between 1000 and 10,000. Scientific notation splits it into a whole part and a fraction:

$$\log(a \times 10^n) = \log a + n$$

Since a is between 1 and 10, $\log a$ is between 0 and 1. So $\log(3.0 \times 10^3) = \log 3.0 + 3 = 0.48 + 3 = 3.48$. The exponent gives the whole number; the coefficient adds the decimal part.

This makes estimating easy. For $\log(3.2 \times 10^{-4})$, the answer is -4 plus something less than 1, so it lies between -4 and -3: it is -3.49. A calculator answer of -4.49 or 3.49 fails that check at once.

7.3 Going back: the antilog

To undo a logarithm, raise 10 to it. This is the **antilog**, the 10^x key on a calculator. If $\log x = 2.70$, then $x = 10^{2.70} = 501$. The whole-number part of the logarithm sets the power of ten, so $10^{2.70}$ is between 10^2 and 10^3, and 501 is.

Antilog
Undoing a logarithm: the antilog of y is 10^y.

In general chemistry the antilog appears most in one place. The **pH** of a solution is defined from its hydrogen ion concentration, written $[H^+]$, in moles per liter:

$$\text{pH} = -\log[H^+] \qquad [H^+] = 10^{-\text{pH}}$$

pH
$\text{pH} = -\log[H^+]$, where $[H^+]$ is the hydrogen ion concentration in mol/L.

You do not need the chemistry of acids for this chapter. You need to move between the two forms without losing the minus sign.

Example 7.2

A solution has pH 3.70. What is $[H^+]$?

What is asked	A concentration, from a pH.
Antilog	$[H^+] = 10^{-3.70}$
Calculate	$10^{-3.70} = 2.0 \times 10^{-4}$ mol/L
Check	-3.70 lies between -4 and -3, so the power of ten is 10^{-4}.
Answer	2.0×10^{-4} mol/L

The antilog undoes the logarithm: $x = 10^{\log x}$. For pH, $[H^+] = 10^{-\text{pH}}$.

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Practice set 7A 8 questions · 12 minutes · calculator allowed

1. What is $\log 0.0001$?

(A) -4 **(B)** -3
(C) 4 **(D)** 0.0001

2. Between which two whole numbers does $\log 4{,}500$ lie?

(A) 2 and 3 **(B)** 3 and 4
(C) 4 and 5 **(D)** 45 and 46

3. If $\log x = 2.30$, what is x?

(A) 0.36 **(B)** 9.97
(C) 200 **(D)** 2.30×10^{2}

4. A solution has $[\mathrm{H}^+] = 1.0 \times 10^{-9}$ mol/L. What is its pH?

(A) -9.00 **(B)** 9.00
(C) 1.00 **(D)** 0.00

5. What is $\log(2.5 \times 10^{-3})$, to the correct number of decimal places?

(A) 2.60 **(B)** -2.6
(C) -3.40 **(D)** -2.60

6. Solution A has pH 4.0. Solution B has 100 times the hydrogen ion concentration of A. What is the pH of B?

(A) 0.040 **(B)** 400
(C) 2.0 **(D)** 6.0

7. A solution has pH 5.30. A student dilutes it so its $[\mathrm{H}^+]$ is one fourth as large. What is the new $[\mathrm{H}^+]$?

(A) 5.0×10^{-6} mol/L **(B)** 1.3×10^{-6} mol/L
(C) 2.0×10^{5} mol/L **(D)** 1.3×10^{-5} mol/L

8. Two solutions have $[\mathrm{H}^+] = 3.0 \times 10^{-3}$ and 3.0×10^{-6} mol/L. Equal volumes are mixed, so the mixture's $[\mathrm{H}^+]$ is the average of the two. What is the mixture's pH?

(A) 2.52 **(B)** 4.02
(C) 2.82 **(D)** 4.50

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8 Graphs and tables

In 1958 Charles David Keeling began measuring carbon dioxide in the air on Mauna Loa, in Hawaii. His graph has run ever since, and it does two things at once. It climbs every year, and it wobbles every year: carbon dioxide falls each northern summer, when the forests of the Northern Hemisphere take it in, and rises each winter. One line holds both stories, and reading it means knowing which one a question is asking about.

8.1 Reading a table

A data table is a set of measurements laid out so each row is one trial and each column is one **variable**. Read the column heads before the numbers. The unit is part of the head, usually in parentheses, and it applies to every number in that column. A column headed "Mass (g)" holding 0.52 means 0.52 g; a column headed "Mass (mg)" holding the same number means a thousand times less.

Variable
A quantity that changes from one row or one point to the next.

Trial	Volume (mL)	Mass (g)	Mass ÷ volume (g/mL)
1	10.0	13.5	1.35
2	20.0	27.1	1.36
3	30.0	40.4	1.35
4	40.0	54.2	1.36

Look for what stays the same. In this table mass and volume both change, but their ratio barely moves: the liquid's density is about $1.35\ \text{g/mL}$ in every trial. A constant ratio between two columns is the signature of a direct proportion, from Chapter 6.

8.2 Reading a graph

By convention the quantity you set, the **independent variable**, goes on the horizontal axis, and the quantity that responds, the **dependent variable**, goes on the vertical axis. Before reading any point, read both axis labels with their units, and then read the scale: how much one grid step is worth. Most misread graphs come from assuming each step is 1 when it is 2, 5 or 0.5.

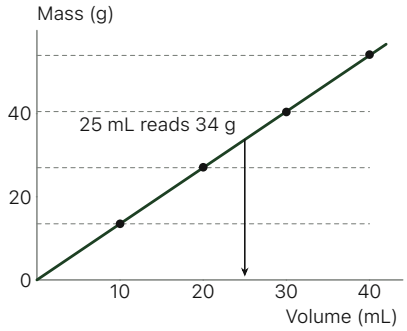
Reading a value between measured points is **interpolation**. Reading beyond the last point, by extending the line, is **extrapolation**, and it is less reliable, because nothing guarantees the pattern continues past the data.

Independent variable
The quantity the experimenter sets. It goes on the horizontal axis.

Dependent variable
The quantity that responds. It goes on the vertical axis.

Interpolation
Reading a value between measured points from the line through them.

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The table's data, with the line through them. Interpolating at 25 mL gives about 34 g.

Read the axis labels, the units and the value of one grid step before you read any point.

8.3 Slope

The **slope** of a straight line is how much the vertical quantity changes for each unit of the horizontal one:

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope
Rise over run, $\frac{\Delta y}{\Delta x}$, with units of the vertical axis over the horizontal.

A slope has units, the vertical axis's unit over the horizontal's, and those units tell you what it means. On a graph of mass in grams against volume in milliliters, the slope is in g/mL: it is the density. On a graph of distance against time it is a speed.

Take the slope from two points on the line, far apart, not from two data points that may sit off it. The farther apart the points, the less a small misreading matters.

The **intercept** is where the line meets the vertical axis. A line through the origin means the two quantities are directly proportional. A line with a nonzero intercept is still straight, but the two quantities are not proportional: $y = mx + b$ with $b \neq 0$. The mass of a beaker and liquid together, against the liquid's volume, is a straight line whose intercept is the mass of the empty beaker.

Intercept
Where a line crosses the vertical axis: the value of y when $x = 0$.

Example 8.1

A line on a graph of mass (g) against volume (mL) passes through $(5.0, 18.0)$ and $(25.0, 42.0)$. What is its slope, what does it mean, and what is the intercept?

What is asked	The slope with units, its meaning, and the value of mass at zero volume.
Slope	$\frac{42.0 - 18.0}{25.0 - 5.0} = \frac{24.0}{20.0} = 1.20\,\text{g/mL}$: the liquid's density.
Intercept	$y = 1.20x + b$ through $(5.0, 18.0)$: $18.0 = 6.0 + b$, so $b = 12.0\,\text{g}$.
Meaning	At zero liquid the mass is 12.0 g: the container.
Check	At 25.0 mL: $1.20 \times 25.0 + 12.0 = 42.0$.
Answer	**Slope 1.20 g/mL, the density; intercept 12.0 g, the container**

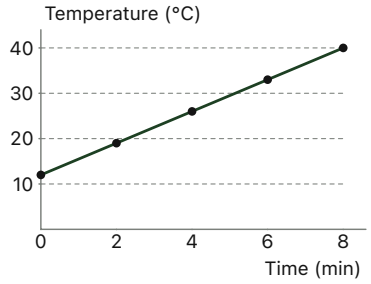
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Practice set 8A 8 questions · 14 minutes · calculator allowed

1. A table has a column headed “Mass (mg)”. One entry is 250. What is that mass in grams?

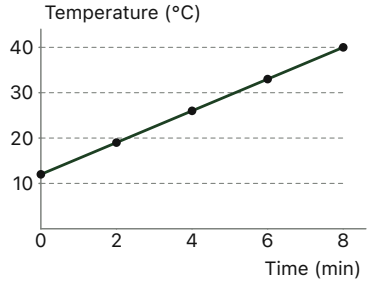
(A) 250 g **(B)** 2.50 g
(C) 0.250 g **(D)** 250,000 g

2. The graph shows the temperature of a liquid as it is heated. What is the temperature at 5 minutes?



(A) 26 °C **(B)** 29.5 °C
(C) 33 °C **(D)** 47 °C

3. For the same graph, what is the slope of the line, with units?

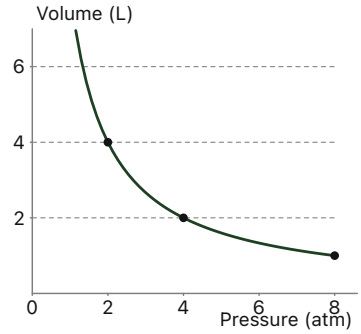


(A) 7.0 °C/min **(B)** 5.0 °C/min
(C) 3.5 °C/min **(D)** 0.29 min/°C

4. A graph of y against x is a straight line that crosses the vertical axis at $y = 5$. Which statement is true?

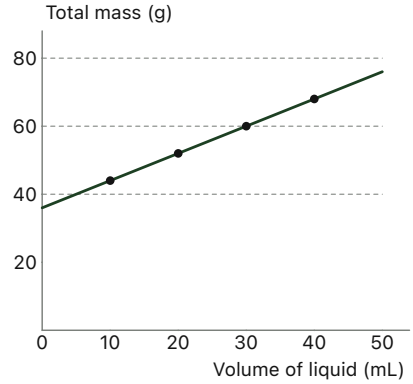
(A) The slope is 5
(B) y changes by a constant amount for each unit of x
(C) y is inversely proportional to x
(D) y is directly proportional to x

5. The graph shows the volume of a gas sample at different pressures. What kind of relationship is it, and what is the volume at 1.0 atm?



(A) Inverse; 4.0 L **(B)** Direct; 2.0 L
(C) Inverse; 8.0 L **(D)** Direct; 0.50 L

6. Liquid is added to a beaker, and the total mass of beaker and liquid is plotted. What is the mass of the empty beaker, and what is the density of the liquid?



(A) 36 g; 0.80 g/mL **(B)** 36 g; 1.25 g/mL
(C) 44 g; 0.80 g/mL **(D)** 0 g; 1.7 g/mL

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9 Your calculator

Type $6 \div 2(1+2)$ into two different calculators and you can get two different answers, 9 and 1. Neither machine is broken. Some treat a multiplication written without a sign as binding more tightly than division, and some do not. A calculator does exactly what it is told in the order its makers chose, and most calculator errors on chemistry exams are not arithmetic errors at all. They are the gap between what you meant and what you typed.

9.1 Order of operations and parentheses

A calculator follows the **order of operations**: parentheses first, then exponents, then multiplication and division from left to right, then addition and subtraction from left to right. It does not see a fraction bar. When you write

$$\frac{2.50 + 1.30}{0.40 \times 5.0}$$

the bar groups the top and the bottom, but typing `2.50 + 1.30 / 0.40 * 5.0` gives $2.50 + (1.30 \div 0.40) \times 5.0 = 18.75$, not 1.9. The fraction bar has to become parentheses: `(2.50 + 1.30) / (0.40 * 5.0)`.

The bottom of a fraction is where this costs the most. $\frac{a}{bc}$ typed as `a / b * c` divides by b and then multiplies by c, putting c on top. Either put the bottom in parentheses, `a / (b * c)`, or divide by each factor in turn, `a / b / c`. Both are right; `a / b * c` never is.

Order of operations
Parentheses, then exponents, then multiplication and division left to right, then addition and subtraction left to right.

Example 9.1

Evaluate $\dfrac{(0.500)(0.0821)(298)}{(1.25)(0.950)}$.

What is asked	One number, from a product over a product.
Type	`0.500 * 0.0821 * 298 / (1.25 * 0.950)`, or `0.500 * 0.0821 * 298 / 1.25 / 0.950`
Result	10.30
Check	Top: about $0.5 \times 0.08 \times 300 = 12$. Bottom: about 1.2. Their ratio is about 10.
Answer	**10.3**

A fraction bar is parentheses around the top and around the bottom. Divide by every factor in the bottom.

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9.2 Scientific notation keys

Enter a number in scientific notation with the **EE key** (EXP on some models, $\times 10^x$ on others). To enter 6.022×10^{23}, type `6.022 EE 23`. The key means "times ten to the," so do not also type $\times 10$: `6.022 * 10 EE 23` enters 6.022×10^{24}, ten times too big.

The EE key also keeps a number in one piece. Dividing by 6.022×10^{23} typed with EE divides by the whole number. Typed with the power key, `x / 6.022 * 10 ^ 23`, the calculator divides by 6.022 and then multiplies by 10^{23}, an error of 10^{46}. If you use the power key, put the whole number in parentheses.

EE key
The key, labeled EE, EXP or $\times 10^x$, that enters the power of ten of a number in scientific notation.

Example 9.2

How many moles are 3.01×10^{21} molecules? There are 6.022×10^{23} in one mole.

What is asked	Moles, by dividing.
Type	`3.01 EE 21 / 6.022 EE 23`
Result	5.00×10^{-3}
Check	The exponents alone give $10^{21-23} = 10^{-2}$, and $\frac{3}{6} = 0.5$, so $0.5 \times 10^{-2} = 5 \times 10^{-3}$.
Answer	5.00×10^{-3} mol

EE means times ten to the. Never type $\times 10$ before it, and never split a number in scientific notation with the power key.

9.3 Negatives and exponents

Many calculators have two minus keys. The subtraction key takes one number from another; the **negation key**, usually $(-)$, makes a single number negative, as in the exponent of 10^{-5}. Using subtraction where negation belongs gives a syntax error on some models and a wrong number on others.

Exponents bind before negation. -3^2 means $-(3^2) = -9$, and $(-3)^2 = 9$. On a calculator, `(-) 3 ^ 2` usually returns -9. When you mean the square of a negative number, use parentheses. The same rule matters for $10^{-\text{pH}}$: type the whole exponent, sign included, after the 10^x key.

Negation key
The key, usually labeled $(-)$, that makes a number negative. It is different from the subtraction key.

Use the negation key for a negative number and the subtraction key to subtract. Wrap a negative base in parentheses before squaring it.

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Practice set 9A 8 questions · 12 minutes · calculator allowed

1. Which keystrokes compute $\dfrac{8.0}{2.0 \times 4.0}$?

(A) 8.0 / 2.0 * 4.0 **(B)** 8.0 / (2.0 * 4.0)
(C) (8.0 / 2.0) * 4.0 **(D)** 8.0 * 2.0 / 4.0

2. A student types 3.0 * 10 EE 8 to enter 3.0×10^8. What number did the calculator receive?

(A) 3.0×10^7 **(B)** 3.0×10^{80}
(C) 3.0×10^9 **(D)** 3.0×10^8

3. What is -4^2, following the order of operations?

(A) 16 **(B)** -16
(C) -8 **(D)** 8

4. A student types 64 ^ 1/3 to find the cube root of 64. What does the calculator return?

(A) 262,144 **(B)** 0.333
(C) 21.3 **(D)** 4

5. Evaluate $\dfrac{(2.50)(8.31)(310)}{(0.750)(4.00)}$.

(A) 2,150 **(B)** 2.15
(C) 3.43 × 10^4 **(D)** 1.94 × 10^4

6. Which is the best estimate of $\dfrac{(6.1 \times 10^{23})(0.0480)}{3.9 \times 10^2}$?

(A) 7.5×10^{18} **(B)** 7.5×10^{25}
(C) 7.5×10^{21} **(D)** 7.5×10^{19}

7. A student finds moles with $n = \frac{PV}{RT}$, where $P = 0.968$, $V = 2.50$, $R = 0.0821$ and $T = 25 + 273$. She types 0.968 * 2.50 / 0.0821 * 25 + 273 and gets 1010. What is the correct value of n?

(A) 0.0907 **(B)** 0.0989
(C) 1.18 **(D)** 1010

8. pH is $-\log[\mathrm{H}^+]$. A solution has $[\mathrm{H}^+] = 2.5 \times 10^{-4}$ mol/L. It is mixed with an equal volume of water, which halves $[\mathrm{H}^+]$. What is the new pH?

(A) -3.90 **(B)** 3.30
(C) 3.60 **(D)** 3.90

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10 Setting up word problems

In 1945 the mathematician George Pólya published a small book called How to Solve It. Its method fits on an index card: understand the problem, make a plan, carry out the plan, look back. It has sold more than a million copies, and the advice has not needed revising, because the hard part of a word problem has never been the arithmetic. It is deciding, before any arithmetic, what the problem is asking and which facts lead there.

10.1 Read for the question

Read the last sentence first. It holds the question, and the question names the **target quantity**: a mass, a volume, a concentration, a percent. Write it down with its unit before anything else, as "? g of copper" or "? mL of solution." Most wrong answers on a first chemistry exam are right answers to a different question: the mass of the solution when the problem asked for the mass of the solute, or milliliters when it asked for liters.

Target quantity
The quantity a problem asks for, with its unit.

Then read the whole problem again and mark the words that change the target: *of*, *per*, *remaining*, *total*, *each*. "The mass of copper in the sample" and "the mass of the sample" differ by one word and one step.

10.2 List what is given

Next, list every **given** as a number, a unit and a label: "25.0 mL solution," "0.150 mol per L," "8.5% tin by mass." The label matters as much as the unit. Two masses in grams, one of solute and one of solution, cannot be told apart by their units, only by their labels, and confusing them is the error that labels prevent.

Given
A quantity stated in the problem, written with its unit and a label.

A given is sometimes hidden in a word. "Pure" means 100%. "Doubled" is a factor of 2. "Room temperature" is about 25 °C. "A dozen" is 12, exactly. And a quantity you need may not be given at all but be a constant: a density you are told to look up, or a conversion such as $1\,\text{L} = 1000\,\text{mL}$.

Write the target with its unit first. Then list every given with a number, a unit and a label.

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10.3 Plan a path

A plan is a route from a given to the target through quantities you can connect. Draw it as a **unit map**: the units in a row, with the factor that links each pair written over the arrow.

Unit map
A chain of units from the given to the target, with the factor that links each pair.

$$\text{mL solution} \xrightarrow{1\ \text{L}=1000\ \text{mL}} \text{L solution} \xrightarrow{0.150\ \text{mol/L}} \text{mol solute} \xrightarrow{58.4\ \text{g/mol}} \text{g solute}$$

Each arrow becomes one conversion factor in a dimensional-analysis line, Chapter 5's method. If you cannot draw an arrow from where you are, look for a given you have not used yet. A given left over at the end usually means the plan skipped a step; a plan that needs a number you were not given usually means a step is wrong.

Many problems are **layered**: the answer to one step is the input to the next. A layered problem is no harder at any single step than a one-step problem. What makes it hard is keeping track of which intermediate number is which. Label every intermediate result, as "35.3 g liquid" rather than "35.3," and carry an extra digit until the end.

Layered problem
A problem in which the result of one step is the input to the next.

Example 10.1

A saline solution is 0.90% sodium chloride by mass and has a density of $1.005\ \text{g/mL}$. What mass of sodium chloride is in 250.0 mL of it?

What is asked	? g sodium chloride. Not the mass of the solution.
Givens	0.90% NaCl by mass; 1.005 g/mL solution; 250.0 mL solution.
Plan	mL solution $\rightarrow$ g solution (density) $\rightarrow$ g NaCl (percent).
Solution	$250.0\ \text{mL} \times \dfrac{1.005\ \text{g}}{1\ \text{mL}} = 251.25\ \text{g solution}$
Solute	$251.25\ \text{g} \times 0.0090 = 2.26\ \text{g NaCl}$
Check	Under 1% of about 250 g should be about 2 g. Two figures, from 0.90%.
Answer	**2.3 g of sodium chloride**

Draw the unit map before calculating. Every arrow is one factor; every given should be used once.

10.4 Look back

Look back at three things. The unit: is it the target's unit? The size: a rough estimate from Chapter 9 should agree within a factor of two or so. The sense: a part cannot exceed its whole, a percent by mass cannot exceed 100, a dilute solution should have a smaller con-

Reasonableness check
Asking whether an answer's size, sign and unit fit the situation.

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Practice set 10A 8 questions · 16 minutes · calculator allowed

1. A problem asks: “What volume of a 12% sugar solution contains 30 g of sugar?” What is the target quantity?

(A) A mass of sugar
(B) A volume of solution
(C) A percent
(D) A mass of solution

2. A sample of brass is 35% zinc by mass. A brass key has a mass of 8.0 g. What mass of copper is in the key, if brass is only copper and zinc?

(A) 23 g **(B)** 8.0 g
(C) 5.2 g **(D)** 2.8 g

3. Which given in this problem is not needed? “A 250 mL bottle of olive oil, density 0.92 g/mL, costs 9 dollars. What is the mass of the oil?”

(A) None; all three are needed
(B) The price
(C) The density
(D) The volume

4. A runner’s pace is 7.5 minutes per mile. How many kilometers does she run in 45 minutes? (1 mi $= 1.609$ km)

(A) 3.7 km **(B)** 6.0 km
(C) 9.7 km **(D)** 540 km

5. A 500.0 mL solution contains 0.250 mol of solute. A student pours out 120.0 mL. How many moles of solute are in the portion she poured?

(A) 0.0600 mol **(B)** 0.190 mol
(C) 0.250 mol **(D)** 1.04 mol

6. A stock solution contains 40.0 g of salt per liter. A student mixes 25.0 mL of it with 75.0 mL of water. What is the concentration of the new solution, in grams per liter?

(A) 40.0 g/L **(B)** 30.0 g/L
(C) 13.3 g/L **(D)** 10.0 g/L

7. A fertilizer is 20.0% nitrogen by mass. A lawn needs 1.5 g of nitrogen per square meter. How many kilograms of fertilizer cover a lawn of 400 m^2?

(A) 0.12 kg **(B)** 0.60 kg
(C) 3.0 kg **(D)** 30 kg

8. A pool holds 60.0 m^3 of water. Chlorine should be 2.0 mg per liter. A chlorine product is 65% chlorine by mass. What mass of the product, in grams, is needed?

(A) 1.8×10^5 g **(B)** 185 g
(C) 120 g **(D)** 78 g

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