1 Fractions, ratios and percents

In the early 1980s A&W sold a third-pound burger to compete with the Quarter Pounder, at the same price. It did not sell. The company's own research found that many customers believed they were being charged the same money for less meat, because 3 is smaller than 4. A fraction is a division, and a bigger number underneath means a smaller piece. Every chemistry formula with a bar in it is built on that one idea.

1.1 A fraction is a division

A fraction is one number divided by another. The bar means divide: $\frac{3}{8}$ is $3 \div 8 = 0.375$. Read every fraction bar in a formula the same way. When you see $d = \frac{m}{V}$, density is mass divided by volume, and when you see a unit such as g/mL, the slash is the same bar: grams divided by milliliters.

Fraction
A division written with a bar: the numerator divided by the denominator.

The word **per** also means divide. Grams per milliliter, moles per liter and miles per hour are all one quantity divided by another. When a problem says "per," put the quantity before it on top and the quantity after it on the bottom.

Per
Divided by. A quantity per unit is a rate.

Simplifying and the reciprocal

To simplify, divide the top and bottom by the same number: $\frac{18}{24} = \frac{3}{4}$, dividing both by 6. This does not change the value, because you have divided by $\frac{6}{6}$, which is 1. That fact, that multiplying or dividing by 1 leaves a value alone, is the whole basis of unit conversion in Chapter 5.

The **reciprocal** of a fraction is the fraction flipped: the reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$, and the reciprocal of 4 is $\frac{1}{4}$. A number times its reciprocal is always 1.

Reciprocal
A fraction turned upside down. A number times its reciprocal is 1.

The four operations

Operation	Rule	Example
Multiply	Top times top, bottom times bottom	$\frac{2}{3} \times \frac{9}{10} = \frac{18}{30} = \frac{3}{5}$
Divide	Multiply by the reciprocal of the second	$\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \times \frac{8}{9} = \frac{2}{3}$
Add or subtract	Rewrite over a common denominator first	$\frac{1}{4} + \frac{1}{6} = \frac{3}{12} + \frac{2}{12} = \frac{5}{12}$

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Adding fractions is the one place students reach for a shortcut that does not exist. $\frac{1}{4} + \frac{1}{6}$ is not $\frac{2}{10}$. Fourths and sixths are pieces of different sizes, so cut them into a size they share, twelfths, and then count.

Example 1.1

Simplify $\dfrac{\frac{5}{6}}{\frac{10}{9}}$.

What is asked	One fraction divided by another, written as a stacked fraction. The long bar is the division.
Rewrite	$\frac{5}{6} \div \frac{10}{9} = \frac{5}{6} \times \frac{9}{10}$
Multiply	$\frac{5 \times 9}{6 \times 10} = \frac{45}{60}$
Simplify	Divide top and bottom by 15: $\frac{3}{4}$.
Check	$\frac{5}{6} \approx 0.833$ and $\frac{10}{9} \approx 1.111$; $0.833 \div 1.111 = 0.75$.
Answer	**$\frac{3}{4}$, or 0.75**

A fraction bar, a slash and the word per all mean divide. To divide by a fraction, multiply by its reciprocal.

1.2 Ratios and rates

A **ratio** compares two amounts. Water has two hydrogen atoms for every oxygen atom, a ratio of $2 : 1$. The ratio does not tell you how much water there is; it tells you how the parts compare, and it holds for one molecule or for a whole lake.

Ratio
A comparison of two quantities by division, written $a : b$ or $\frac{a}{b}$.

A ratio of parts also gives you each part's share of the whole. In a ratio of $2 : 1$ there are $2 + 1 = 3$ parts in all, so hydrogen atoms are $\frac{2}{3}$ of the atoms in water and oxygen atoms are $\frac{1}{3}$. Students often write $\frac{2}{1}$ when the question asks for a fraction of the whole. Add the parts first.

A **rate** is a ratio of two different kinds of quantity. A **unit rate** is a rate scaled so that the second quantity is 1: 45.0 g in 50.0 mL is $\frac{45.0}{50.0} = 0.900$ g per 1 mL. A unit rate is the most

Rate
A ratio of two quantities with different units, such as grams per milliliter.

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useful form of a rate, because one multiplication then answers any question about another amount.

Example 1.2

A cleaning solution is made by mixing concentrate and water in a ratio of $3 : 5$ by volume. How many milliliters of concentrate are in 240 mL of the finished solution?

What is asked	The volume of one part, concentrate, in a known total. Not the volume of water.
Total parts	$3 + 5 = 8$ parts, so concentrate is $\frac{3}{8}$ of the whole.
Multiply	$\frac{3}{8} \times 240\,\text{mL} = 90\,\text{mL}$
Check	Water is then $240 - 90 = 150$ mL, and $90 : 150$ reduces to $3 : 5$.
Answer	**90 mL**

To find a part from a ratio, add the parts to get the whole, then take that part's share of the whole.

1.3 Percents

A **percent** is a fraction with 100 underneath: 35% is $\frac{35}{100}$, or 0.35. To use a percent in a calculation, move the decimal point two places left and multiply. To turn a decimal into a percent, move it two places right.

Percent
Per hundred. 35% means $\frac{35}{100}$, or 0.35.

Every percent question is one equation with three pieces:

$$\text{part} = \text{fraction} \times \text{whole}$$

Find which two the question gives you, and solve for the third. The whole is the quantity the percent is "of." In "12% of the sample is salt," the whole is the sample.

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Chemistry uses percent most often as **percent by mass**: the mass of one part divided by the mass of the whole, times 100. A 250.0 g sample that contains 40.0 g of sugar is $\frac{40.0}{250.0} \times 100 = 16.0\%$ sugar by mass.

Percent by mass
The mass of one component divided by the total mass, times 100.

Example 1.3

An alloy is 8.5% tin by mass. What mass of the alloy contains 34.0 g of tin?

What is asked The whole: the mass of alloy. The 34.0 g of tin is the part.

Equation part = fraction × whole, so $34.0\,\text{g} = 0.085 \times \text{whole}$.

Solve $\text{whole} = \dfrac{34.0\,\text{g}}{0.085} = 400\,\text{g}$

Check The whole must be much bigger than the part, since tin is under a tenth of it. 8.5% of 400 g is 34 g.

Answer **400 g of alloy**

Part equals fraction times whole. Name the whole first: it is the quantity the percent is of.

1.4 Percent change and percent error

A **percent change** compares a change with where you started:

$$\text{percent change} = \frac{\text{new} - \text{old}}{\text{old}} \times 100$$

Percent change
$\frac{\text{new} - \text{old}}{\text{old}} \times 100$. Positive for an increase, negative for a decrease.

Divide by the old value, never the new one. A price that goes from \$40 to \$50 rose by $\frac{10}{40}$, or 25%. Going back from \$50 to \$40 is a drop of $\frac{10}{50}$, or 20%. The same \$10 is a different percent because the starting point differs.

In a laboratory you compare a measurement with an accepted value using **percent error**. It is the size of the miss divided by the accepted value:

$$\text{percent error} = \frac{|\,\text{measured} - \text{accepted}\,|}{\text{accepted}} \times 100$$

Percent error
$\frac{|\text{measured} - \text{accepted}|}{\text{accepted}} \times 100$.

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The bars mean the absolute value, so percent error is never negative. The accepted value goes underneath, in the same place the old value goes in a percent change.

Example 1.4

A student measures the density of aluminum as $2.81\ \text{g/mL}$. The accepted value is $2.70\ \text{g/mL}$. What is the percent error?

What is asked Percent error: the miss as a percent of the accepted value.

Miss $|2.81 - 2.70| = 0.11\ \text{g/mL}$

Divide $\dfrac{0.11}{2.70} \times 100 = 4.1\%$

Check The miss is a little more than a tenth of a gram per milliliter on a value near 3, so a few percent is the right size. Dividing by 2.81 instead would give 3.9%.

Answer **4.1%**

A percent change divides by the old value. A percent error divides by the accepted value and is never negative.

Common slips

The slip	The fix
Adding fractions straight across	Rewrite over a common denominator, then add the numerators only.
Dividing by a fraction without flipping it	Multiply by the reciprocal of the second fraction.
Writing $\frac{2}{1}$ for "what fraction is hydrogen"	Add the parts of the ratio to get the whole, then divide.
Using 35 in place of 0.35	Move the decimal two places left before you multiply by a percent.
Dividing a percent change by the new value	Always divide by the starting value.
Treating the part as the whole	Ask what the percent is *of*. That is the whole.

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Practice set 1A 8 questions · 12 minutes · no calculator needed

1. What is $\frac{3}{4} \div \frac{3}{8}$?

(A) $\frac{9}{32}$ **(B)** $\frac{1}{2}$
(C) 2 **(D)** $\frac{32}{9}$

2. What is $\frac{1}{4} + \frac{2}{3}$?

(A) $\frac{3}{7}$ **(B)** $\frac{11}{12}$
(C) $\frac{2}{12}$ **(D)** $\frac{3}{12}$

3. A molecule has 2 carbon atoms, 6 hydrogen atoms and 1 oxygen atom. What fraction of its atoms are hydrogen?

(A) 3 **(B)** $\frac{3}{4}$
(C) $\frac{2}{3}$ **(D)** $\frac{1}{3}$

4. A solution is made from 12 g of salt and enough water to make 80 g of solution. What is the percent by mass of salt?

(A) 13% **(B)** 15%
(C) 18% **(D)** 85%

5. The mass of a sample rises from 40 g to 50 g. What is the percent increase?

(A) 10% **(B)** 20%
(C) 25% **(D)** 125%

6. Brine is made by mixing salt and water in a mass ratio of $1 : 9$. How much salt is in 150 g of brine, and what percent of the brine is salt?

(A) 15 g; 10% **(B)** 16.7 g; 11.1%
(C) 15 g; 11.1% **(D)** 135 g; 90%

7. An ore is 4.0% copper by mass. A refinery processes 2.5 kg of ore and recovers 90% of the copper in it. What mass of copper is recovered?

(A) 9.0 g **(B)** 90 g
(C) 100 g **(D)** 2.3 kg

8. The accepted density of aluminum is 2.70 g/mL. A student's first measurement is 2.43 g/mL. Her second measurement is also low and has half the percent error of the first. What is her second measurement?

(A) 2.65 g/mL **(B)** 2.31 g/mL
(C) 2.84 g/mL **(D)** 2.57 g/mL

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Practice set 1B 8 questions · 12 minutes · calculator allowed

1. A liquid has a mass of 36.0 g and a volume of 45.0 mL. What is its mass per milliliter?

(A) 0.800 g/mL **(B)** 1.25 g/mL
(C) 81.0 g/mL **(D)** 0.800 mL/g

2. What is $\dfrac{2/5}{4/15}$?

(A) $\frac{8}{75}$ **(B)** $\frac{2}{3}$
(C) $\frac{3}{2}$ **(D)** $\frac{15}{2}$

3. A mixture is 3 parts ethanol to 7 parts water by volume. How many milliliters of water are in 450 mL of the mixture?

(A) 1050 mL **(B)** 315 mL
(C) 193 mL **(D)** 135 mL

4. A 2.40 kg bag of fertilizer is 15% nitrogen by mass. What mass of nitrogen does it contain?

(A) 2.04 kg **(B)** 160 g
(C) 36.0 g **(D)** 360 g

5. On cooling, the volume of a gas sample falls from 250 mL to 200 mL. What is the percent change in volume?

(A) -25% **(B)** -20%
(C) $+20\%$ **(D)** -80%

6. A student needs 18.0 g of sugar. Her stock mixture is 40.0% sugar by mass, and she has 60.0 g of it. What percent of her stock mixture must she use?

(A) 30.0% **(B)** 45.0%
(C) 75.0% **(D)** 12.0%

7. A 75.0 g mixture contains substances X and Y in a mass ratio of $2:3$. Then 10.0 g of Y is removed. What percent of the new mixture is X?

(A) 40.0% **(B)** 46.2%
(C) 30.8% **(D)** 53.8%

8. A student measures a melting point of 79.2 °C. The accepted value is 80.3 °C. She then learns her thermometer reads 0.6 °C low and corrects her reading. What is the percent error of the corrected value?

(A) 0.62% **(B)** 1.37%
(C) 2.12% **(D)** 0.75%

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